ELASTIC CHARACTERISTICS OF ELASTOMERS

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Two characteristics are introduced for elastomers subjected to uniaxial tension: Poisson's ratio ν and the initial modulus E_{tr}. For ten types of rubbers employed in shoes [1], these quantities are constant right up to fracture deformations. It is shown that the shear modulus G, Young's modulus E, and the bulk modulus K of rubbers can be calculated from the known value of E_{tr} and ν. The coefficients E_{tr}, ν, G, K, and E (which are constant for large elastic (highly elastic) deformations of rubbers) are calculated for deformations of one type – uniaxial tension – in contrast to the coefficients in linear theories.

The coefficients in linear theories (Poisson's ratio $\bar{\nu}$, Young's modulus $\bar{E}$, the shear modulus $\bar{G}$, and the bulk modulus $\bar{K}$) are functions of the deformations, they are expressed in terms of the constants E_{tr}, ν, G, K, and E, and for small deformations they are equal to them.

Statistical analysis of uniaxial tensile tests for ten types of rubbers [1], showed that the tensile stress–strain curves for rubbers in the coordinates true stress σ_{tr} – relative elongation $(\lambda - 1)$, where λ is the stretching factor, are linear:

$$\sigma_{tr} = E_{tr}(\lambda - 1). \tag{1}$$

The slope E_{tr} of the straight lines $\sigma_{tr} = f(\lambda - 1)$, being constant for each rubber, is the initial modulus. Equation (1) was proposed in [2] as an equation of the highly elastic state.

The curve $\sigma_{tr} = f(\lambda - 1)$, in contrast to the conventional curve $\sigma_0 = f(\lambda - 1)$ that is usually employed, takes into account the change in the transverse cross-sectional area of the sample

$$\sigma_{tr} = \sigma_0 S_0/S \tag{2}$$

where σ_0 is the conventional stress. The areas S_0 and S of the transverse cross section of the sample before and after deformation are related by the relation [1]

$$S_0/S = 1 + 2\nu(\lambda - 1), \tag{3}$$

where ν is a constant for each rubber. The relative change in the volume of the sample is

$$\theta = \frac{(1 - 2\nu)(\lambda - 1)}{1 + 2\nu(\lambda - 1)}. \tag{4}$$

From Eqs. (1)-(3) we obtain

$$\sigma_0 = \frac{E_{tr}(\lambda - 1)}{1 + 2\nu(\lambda - 1)} \tag{5}$$

or

$$\sigma_0 = \frac{-\frac{E_{tr}}{(2\nu)^2}}{(\lambda - 1) + \frac{1}{2\nu}} + \frac{E_{tr}}{2\nu}. \tag{6}$$

Figure 1 shows the characteristic tensile stress-strain curves for rubbers [1] in the coordinates conventional stress σ_0 vs the relative elongation $(\lambda - 1)$ and true stress σ_{tr} vs relative elongation $(\lambda - 1)$ (lines 1 and 2). In accordance with expression (6) the conventional curve approaches asymptotically the straight line CD, whose vertical coordinate is $E_{tr}/2\nu$. The straight line CD is intersected by the curve σ_{tr} vs $(\lambda - 1)$ with $(\lambda - 1) = 1/2\nu$ at the point C. On the conventional diagram the point C corresponds to E (for $\nu = 0$). The moduli E_{tr} and E are related by the relation [3]

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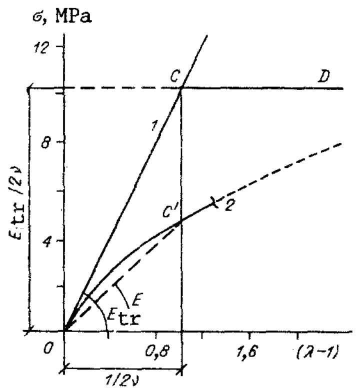

Fig. 1

$$E_{tr} = [1 + 2\nu(\lambda - 1)]E. \tag{7}$$

For small deformations ($\lambda \approx 1$) $E_{tr} = E$ and for large deformations $E_{tr} = E$ only when the transverse cross-sectional area of the sample does not change under uniaxial tension ($\nu = 0$). In this case the curves σ_{tr} vs $(\lambda - 1)$ and σ_0 vs $(\lambda - 1)$ are linear and coincide with one another.

The transverse cross-sectional area of the rubbers investigated changes under tension [1, 2] ($\nu = 0.30$-0.49). As a result, $E_{tr} \neq E$. The transverse cross-sectional area of the uncompressed material ($\nu = 0.5$) changes the most. In contrast to the well-known expression

$$\bar{G} = E/2(1 + \bar{\nu})$$

in the model of an incompressible body the shear modulus is

$$G = E_{tr} = E\lambda. \tag{8}$$

The conventional stress can be represented, from Eqs. (4) and (5), as

$$\sigma_0 = E_{tr}\,\theta/(1 - 2\nu).$$

Here

$$K = E_{tr}/(1 - 2\nu) \tag{9}$$

characterizes the resistance of the rubber to volume deformation.

Expression (9) can be put into the following form with the help of Eq. (7):

$$K = E[1 + 2\nu(\lambda - 1)]/(1 - 2\nu). \tag{10}$$

Hence, $K = \infty$, for $\nu = 0.5$ (for incompressible material) and $K = E$ for $\nu = 0$. The last equality was noted in [4] in the case of compression of a two-dimensional simulator, when the change in the transverse cross section of the deformed rubber is prevented by design.

For small deformations $E_{tr} = E$, $\nu = \bar{\nu}$ and expression (10) becomes the well-known relation

$$K = E/(1 - 2\bar{\nu}).$$

From Eqs. (8) and (10) the ratio of the shear modulus to the bulk modulus for large deformations of the rubbers has the form

$$G/K = (1 - 2\nu)\lambda/[1 + 2\nu(\lambda - 1)] \tag{11}$$

in contrast to the well-known expression

$$\bar{G}/\bar{K} = (1 - 2\bar{\nu})/2(1 + \bar{\nu}). \tag{12}$$

It follows from Eqs. (11) and (12) that if the volume remains constant under uniaxial tension ($\bar{\nu} = \nu = 0.5$), the ratio of the shear modulus to the bulk modulus is equal to zero for large and small deformations.

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NUMERICAL INVESTIGATION OF THE INTERACTION OF A PLANE WAVE WITH A MULTILAYERED CYLINDER IN THE GROUND

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We consider the two-dimensional nonstationary interaction problem of an intense compressional plane wave with an infinitely long multilayered deformable cylinder in the ground with account of elastic and plastic deformations. The rheology of the medium and of the cylinder materials is described by the equations of deformation theory [1] of elastoplastic bodies. In this case one uses as ground distortion function the generalized experimental dependence $\sigma_i = \sigma_i(\varepsilon, \varepsilon_i)$ (ε, ε_i, σ, σ_i are the first and second invariants of the deformation and stress tensors), taking into account the effect of bulk deformation on the nature of plasticity conditions [2] $\sigma_i = \sigma_i(\varepsilon_i)$, and simultaneously with the ground compression diagram $\sigma = \sigma(\varepsilon)$ satisfying in this specific case the sufficient conditions of uniqueness theorems and of minimum work of internal forces, obtained in [3] for a nonlinear medium. A numerical solution of the problem for small and finite deformations of the system investigated is implemented by a difference method of the crossing type [4] in Lagrangian variables without explicit separation of surface discontinuities. The approach mentioned has been used for numerical solution of two-dimensional collision problems of axially symmetric bodies with various obstacles, such as in [5-7].

In the present study specific numerical calculations of the problem are carried out for the case of streamlining of a wave of given intensity around two-layered and three-layered cylinders in the ground with account of wave diffraction by the external surface of the cylinder and of the nonlinearity (including linearity) of its deformable material. We investigate the effects of inelastic properties of the ground, physicomechanical characteristics, and the thickness of cylinder layers on the distributions of kinematic parameters and stresses in them. A comparison is carried out with the stress states of an elastic medium, generated during wave diffraction by a cylindrical cavity. We note that problems related to diffraction of elastic waves by cavities, solids in the presence of elastic fillers, and shells of various shapes in an unbounded elastic or acoustic medium, were treated in [8-12].

The present study is an extension of [13, 14] in the study of characteristic features of plane wave interactions with a multilayered cylinder in the ground and the behavior of its parameters under strong action.

Let the front of the intense plane wave propagating in the ground at the moment of time $t = 0$ be adjacent to the external surface of a long two- or three-layered cylinder in the ground. For a given wave intensity it is necessary to determine for $t > 0$ the stress-deformation states and the kinematic parameters of the ground and of the cylinder with account of wave diffraction, the cylinder material deformability, and the elastoplastic deformations generated in this case.

Since the problem is solved within the two-dimensional statement, the equations of motion of the ground and of the ring-shaped element of the cylinder are in the Lagrange variables (r, φ)

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